## Equations for Calculating Reference Crop ET from Hourly Weather Data

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## Reference Crop ET by the FAO-56 Method

Reference crop evapotranspiration $\left(\mathrm{ET}_{0}\right)$ can be estimated on an hourly basis using the Penman-Monteith equation (Allen, 2000)

$$
\begin{equation*}
E T_{0}=\frac{0.408 \Delta\left(R_{n}-G\right)+\gamma \frac{37}{T+273.2} u_{2}\left(e_{s}-e_{a}\right)}{\Delta+\gamma\left(1+0.34 u_{2}\right)} \tag{1}
\end{equation*}
$$

where
$\mathrm{ET}_{0} \quad$ Reference evapotranspiration ( $\mathrm{mm} \mathrm{h}^{-1}$ )
$\mathrm{Rn} \quad$ Net radiation ( $\mathrm{MJ} \mathrm{m}^{-2} \mathrm{~h}^{-1}$ )
G Soil heat flux ( $\mathrm{MJ} \mathrm{m} \mathrm{m}^{-2} \mathrm{~h}^{-1}$ )
T Air temperature (C)
$\mathrm{e}_{\mathrm{s}} \quad$ saturation vapor pressure at air temperature ( kPa )
$\mathrm{e}_{\mathrm{a}} \quad$ vapor pressure of air (kPa)
$\mathrm{u}_{2} \quad$ Wind speed at $2 \mathrm{~m}\left(\mathrm{~m} \mathrm{~s}^{-1}\right)$
$\Delta \quad$ slope of saturation vapor pressure curve at air temperature $\left(\mathrm{kPa} \mathrm{C}^{-1}\right)$
$\gamma \quad$ psychrometer constant $\left(\mathrm{kPa} \mathrm{C}^{-1}\right)$

Equation 1 is an estimate of ET from a hypothetical short grass with a height of 0.12 m , a surface resistance of $70 \mathrm{~s} \mathrm{~m}^{-1}$, and a albedo of 0.23 (Allen et al., 1998; Allen, 2000)

## Supporting Calculations

Saturation vapor pressure, $\mathrm{e}_{\mathrm{s}}$, in kPa can be approximated at temperature, T , in C , using the equation of Murray (1967)

$$
\begin{equation*}
e_{s}=0.61078 \exp \left(\frac{17.269 T}{237.3+T}\right) \tag{2}
\end{equation*}
$$

Actual vapor pressure of the air, $\mathrm{e}_{\mathrm{a}}$, in kPa , is the product of the $\mathrm{e}_{\mathrm{s}}$ at air temperature and a simultaneous, collocated measurement of relative humidity (RH): $e_{a}=e_{s} R H$, where RH is between 0 and 1 .

The slope of the saturation vapor pressure curve, $\Delta$, in $\mathrm{kPa} \mathrm{K}^{-1}$, can be calculated as the partial derivative of Muray's Eq.
$\Delta=e_{s}\left(\frac{17.269}{237.3+T}\right)\left(1-\frac{T}{237.3+T}\right)$
noting that $\mathrm{e}_{\mathrm{s}}$ is the result from equation 2.
Atmospheric pressure, P , in kPa , can be approximated from altitude, A , in m , and air temperature, T , in C , as
$P=101.3 \exp \left(\frac{-3.42 \times 10^{-2} A}{T+273.15}\right)$
Pressure can be estimated solely from altitude as
$P=101.3\left(\frac{293-0.0065 A}{293}\right)^{5.26}$
The latent heat of vaporization, L , in $\mathrm{J} \mathrm{kg}^{-1}$, can be approximated as
$L=2.5005 \times 10^{6}-2.359 \times 10^{3}\left(T_{a}+273.15\right)$
Heat capacity of air, $\mathrm{c}_{\mathrm{p}}$, in $\mathrm{J} \mathrm{kg} \mathrm{K}^{-1}$, can be expressed as

$$
\begin{equation*}
c_{p}=1004.7\left(\frac{0.522 e_{a}}{P}+1\right) \tag{6}
\end{equation*}
$$

where $\mathrm{R}_{\mathrm{d}}$ is the gas constant ( $287.04 \mathrm{~J} \mathrm{~kg} \mathrm{~K}^{-1}$ ). The psychrometric constant, $\gamma$, in $\mathrm{kPa}^{-1}$, can be approximated as
$\gamma=\frac{1.61 c_{p} P}{L}$

## References

Allen, R.G., Pereira, L.S., Raes, D., Smith, M. 1998. Crop evapotranspiration: Guidelines for computing crop requirements. Irrigation and Drainage Paper No. 56, FAO, Rome, Italy, 300 pp.

Allen, R.G. 2000. Using the FAO-56 dual crop coefficient method over an irrigated region as part of an evapotranspiration intercomparison study. J. Hydrology 229:27-41.

Murray, F.W. 1967. On the computation of saturation vapor pressure. J. Appl. Meteorol. 6:203204.

Penman, H.L. 1948. Evaporation from open water, bare soil, and grass. Proc. Roy. Soc. London A193:120-146.

## Example ET $\mathbf{o}_{0}$ Calculations for the Konza Prairie Research Natural Area, Manhattan, KS

Example Input Data (hourly weather data)
Global Irradiance, Rs: $\quad 700 \mathrm{~W} \mathrm{~m}^{-2}$
Air Temperature, T ( 1.5 m ): $\quad 30 \mathrm{C}$
Relative Humidity, RH ( 1.5 m ): $\quad 0.4$
Wind Speed, u ( 3 m ): $5 \mathrm{~m} \mathrm{~s}^{-1}$

1. Estimate $\mathrm{R}_{\underline{n}}$ and G

For vegetated surfaces $R_{n}$, in MJ m${ }^{-2}$ hr $^{-1}$ can be estimated as
$\mathrm{Rn}=(0.0036) *[0.76 * \mathrm{Rs}-38.5] \quad$ \{equation based on field measurements from KNRPA watershed 1D)
$\mathrm{Rn}=(0.0036) *(0.76 * 700-38.5)$
$\mathbf{R n}=\mathbf{1 . 7 8} \mathbf{~ m m ~ h}^{\mathbf{- 1}}$
G is assumed to be $0.1 * \mathrm{Rn}$ during the day and $0.5 * \mathrm{Rn}$ during the night
If computing with software, use an if-then statement,
If Rs>0 then $\mathrm{G}=0.1 * \mathrm{Rn}$ else $\mathrm{G}=0.5 * \mathrm{Rn}$
$G=0.1 * 1.78=0.178 \mathrm{~mm} \mathrm{~h}^{-1}$
2. Estimate the vapor pressure deficit $\left(\mathrm{e}_{\underline{s}}-\mathrm{e}_{\mathrm{a}}\right)$

Calculate $\mathrm{e}_{\mathrm{s}}$ first
From Eq. 2, $\mathrm{e}_{\mathrm{s}}$ at 30 C is 4.24 kPa
then
$\mathrm{e}_{\mathrm{s}}-\mathrm{e}_{\mathrm{a}}=\mathrm{e}_{\mathrm{s}}{ }^{*}(1-\mathrm{RH})=4.24^{*}(1-0.4)=2.55 \mathrm{kPa}$
3. Estimate wind speed at 2 m

Most weather stations measure wind speed at 3 m . Winds speed at 2 m can be estimated by assuming a logarithmic wind profile (surface similarity theory, $\mathrm{z}_{\mathrm{o}}=0.015 \mathrm{~m}, \mathrm{~h}=0.08 \mathrm{~m}$ ).
$\mathrm{u}_{2}=\mathrm{u}_{3} * 0.92$
$\mathrm{u}_{2}=5 * 0.92=4.6 \mathrm{~ms}^{-1}$

## 4. Calculate $\Delta$ and $\gamma$

Given an air 30 C air temperature, the result from Eq. 3 is $\boldsymbol{\Delta}=\mathbf{0 . 2 4 3} \mathbf{~ k P a ~ C}{ }^{\mathbf{- 1}}$
Equation 7 is often simplified to the form
$\gamma=0.665 \mathrm{E}-3 * \mathrm{P}$
Equation 4 b yields $\mathrm{P}=96.7 \mathrm{kPa}$ (Assuming $\mathrm{A}=400 \mathrm{~m}$ )
and
$\gamma=0.665 \mathrm{E}-3 * 96.7=0.064 \mathrm{kPa} \mathrm{C}^{-1}$

## 5. Calculate ET

Substituting the above-stated results into Eq. 1, yields
ET $_{0}=\mathbf{0 . 6 1 5} \mathrm{mm} \mathrm{h}^{\mathbf{- 1}}$

